47/100 Sekhar

Minor Exam # 1

Time allowed: 60 mins

ELL 319: Digital Signal Processing (Fall 2015)

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Entry No: 2012EE50563

1. Let $X(e^{j\omega})$ and $Y(e^{j\omega})$ denote the DTFT of sequences $x[n]$ and $y[n]$, respectively. If $Y(e^{j\omega})=\frac{1}{2}\{X(e^{j\omega/2})+X(-e^{j\omega/2})\}$, then determine $y[n]$ (in terms of $x[n]$). (15 points)

$x[n] \longleftrightarrow X(e^{j\omega})$

$x[2n] \longleftrightarrow X(e^{j\omega/2})$

$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$

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$X(-e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\,(-1)^{-n}\, e^{-j\omega n}$

$(-1)^n x[n] \longleftrightarrow X(-e^{+j\omega})$

$y[n] = \dfrac{x[2n] + (-1)^{-2n} x[2n]}{2}$

$\boxed{y[n] = x[2n]}$ How?

3·5−0·9

2. A continuous-time signal $x_a(t)$ is composed of a linear combination of sinusoidal signals of frequencies 300 Hz, 500 Hz, 1.2 kHz, 2.15 kHz, and 3.5 kHz. $x_a(t)$ is sampled at 3 kHz and the sampled sequence is passed through an ideal low-pass filter with a cutoff frequency of 900 Hz, generating $y_a(t)$. What are the frequency components present in the reconstructed signal $y_a(t)$? (15 points)

3, 2.15, .75

for $\Omega_N = 3.5$ $\frac{2.6}{3} \le k \le \frac{4.4}{3}$ $\boxed{k=1}$, ± 0.5 kHz will be av.

Consider one freq. say $\pm\Omega_N$, then all the freq. of form $k\Omega_S - \Omega_N$ and $k\Omega_S + \Omega_N$ will be in sampled sequence.
If we filter using 0.9 KHz filter, then all the freq. in that band will pass.

i.e. $|k\Omega_S - \Omega_N| \le 0.9$ KHz and $|p\Omega_S + \Omega_N| \le 0.9$ kHz

$k = 0, \pm1, \pm2, \ldots$ $p = 0, \pm1, \pm2, \ldots$

2.15, 0.9, 1.25

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$\Omega_N - 0.9 \le k\Omega_S \le \Omega_N + 0.9$

$-\Omega_N - 0.9 \le p\Omega_S \le -\Omega_N + 0.9$

$\Omega_S = 3$ kHz

$\frac{\Omega_N}{3} - 0.3 \le k \le \frac{\Omega_N}{3} + 0.3$

Available freq

$\frac{-\Omega_N}{3} - 0.3 \le p \le \frac{-\Omega_N}{3} + 0.3$

for $\Omega_N = 0.3$: $-0.2 \le k \le 0.4$ $\boxed{k=0}$ | ± 0.3 kHz | $-0.4 \le p \le 0.2$ $\boxed{p=0}$

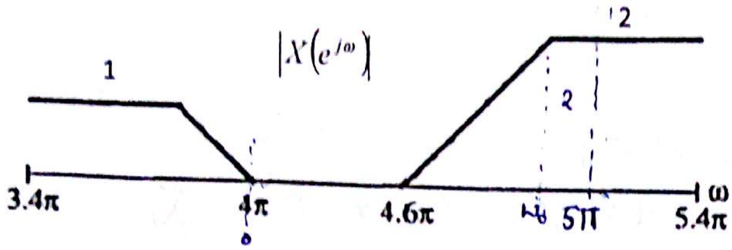
for $\Omega_N = 0.5$: $\frac{-0.4}{3} \le k \le \frac{1.4}{3}$ $\boxed{k=0}$ | ± 0.5 kHz | $\frac{-1.4}{3} \le p \le \frac{+0.4}{3}$ $\boxed{p=0}$

for $\Omega_N = 1.2$: $0.1 \le k \le 0.7$ not poss | | $-0.7 \le p \le -0.1$

for $\Omega_N = 2.15$: $\frac{1.25}{3} \le k \le \frac{3.15}{3}$ $\boxed{k=1}$ | ± 0.75 kHz, ± 0.5 kHz | $-\frac{3.15}{3} \le p \le -\frac{1.25}{3}$ $\boxed{p=-1}$

Scanned by CamScanner

3. The magnitude of $X(e^{j\omega})$, DTFT of a sequence is shown in the figure below for a portion of the ω-axis. Sketch $|X(e^{j\omega})|$ for the frequency range $-\pi \le \omega < \pi$. What type of sequence is $x[n]$? (15 points)



$X(e^{jw})$ is periodic ~~as~~ over any interval of 2π

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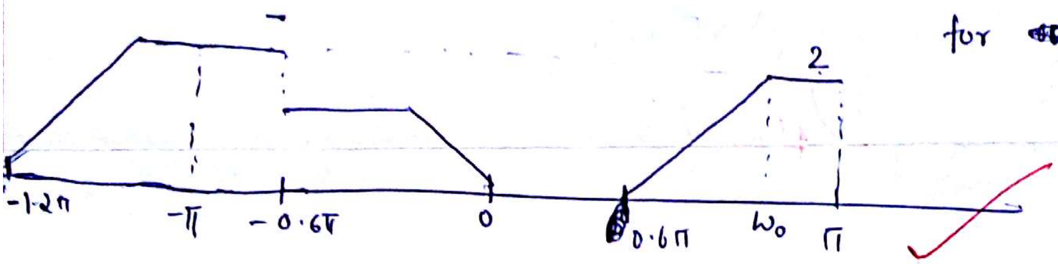
$X(e^{j0}) = X(e^{j4\pi})$

~~$X(e^{jw})$~~ ~~for~~ $X(e^{jw})$ is same ~~as~~ for $4\pi < \omega < 5\pi$ and for $0 < \omega < \pi$

for ~~...~~ $-0.6\pi < \omega < 0$ is same as for $3.4\pi < \omega < 4\pi$



4. A measure of the time delay of a sequence $x[n]$ is given by its centre of gravity: $C_g = \dfrac{\sum_{n=-\infty}^{\infty} n x[n]}{\sum_{n=-\infty}^{\infty} x[n]}$.

Express C_g in terms of $X(z)$. Determine the centre of gravity of the sequence $x[n] = \alpha^n u[n], |\alpha| < 1$. (15 points)

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$$C_g \; \frac{\sum_{n=-\infty}^{\infty} n x[n]}{\sum_{n=-\infty}^{\infty} x[n]}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$X(1) = \sum_{n=-\infty}^{\infty} x[n]$$

$$x[n] \leftrightarrow X(z)$$

$$n x[n] \leftrightarrow -z\frac{dX(z)}{dz}$$

$$\sum_{n=-\infty}^{\infty} n x[n] \leftrightarrow -z\frac{dX(z)}{dz}\Big|_{z=1}$$

$$C_g = \frac{-\dfrac{dX(z)}{dz}}{X(1)}$$

If $x[n] = \alpha^n u[n]$ $|\alpha| < 1$

$$X(z) = \frac{1}{1-\alpha z^{-1}} \quad ROC = |z| > |\alpha|$$

$$-z\frac{dX(z)}{dz} = +z\left(\frac{+1}{(1-\alpha z^{-1})^2}\right)(+\alpha z^{-2}) = \frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$$

$$C_g = \frac{\alpha}{\dfrac{1}{1-\alpha}}$$

$$= \alpha(1-\alpha)$$

5. We wish to compute the autocorrelation function of an upsampled signal as shown in Figure 1 below. It is suggested that this can equivalently be accomplished with the system of Figure 2. Can $H_2(e^{j\omega})$ be chosen so that $\phi_3[m] = \phi_1[m]$? If not, why not? If so, specify $H_2(e^{j\omega})$. (20 points)

Figure 1

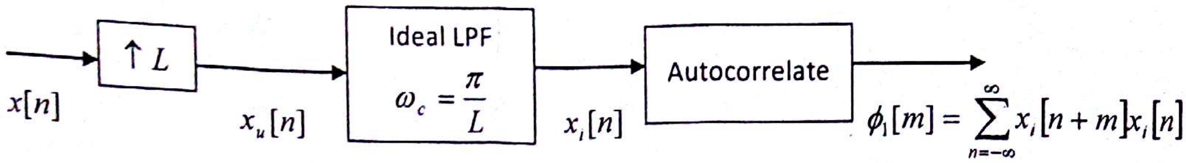
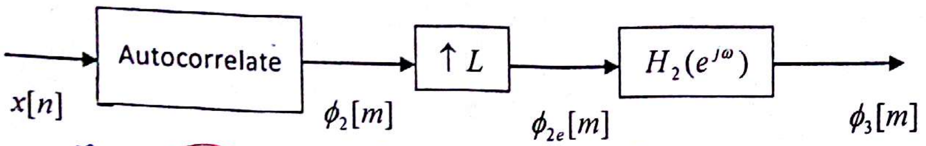


Figure 2



$$x_i[n] = \sum_{k=-\infty}^{\infty} x[k] \underbrace{\frac{\sin[\pi(n-kL)/L]}{\pi(n-kL)/L}}_{y_k[n]}$$

$$\phi_1(m) = \sum_{n=-\infty}^{\infty}\left[\left(\sum_{k=-\infty}^{\infty} x[k]\, y_k[n+m]\right)\left(\sum_{k=-\infty}^{\infty} x[k]\, y_k[n]\right)\right]$$

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$\cancel{\phi_2(m) = \sum_{n=-\infty}^{\infty}}$

$$\phi_{2e}[m] = \sum_{k=-\infty}^{\infty} \phi_2[k]\, \delta[m-kL]$$

$$\phi_3[m] = \sum_{k=-\infty}^{\infty} \phi_2[k]\, h_2[m-kL]$$

6. In the figure on the next page, a sequence $x[n]$ is filtered separately by three different digital filters $H_a(e^{j\omega})$, $H_b(e^{j\omega})$, and $H_c(e^{j\omega})$, to produce three output sequences which are shown in the figure. The frequency responses of the three digital filters are also shown.

For each of the three output sequences, determine the filter that produced it. Justify your answer for credit. Answers without justification will receive no credit. Justifications can (and should) be brief (one or two sentences in each case). (20 points)

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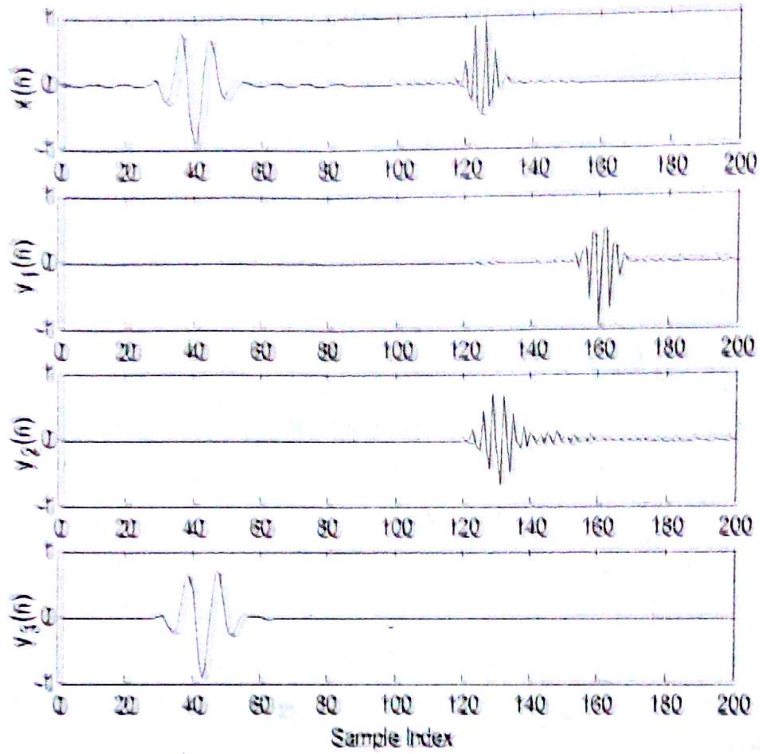
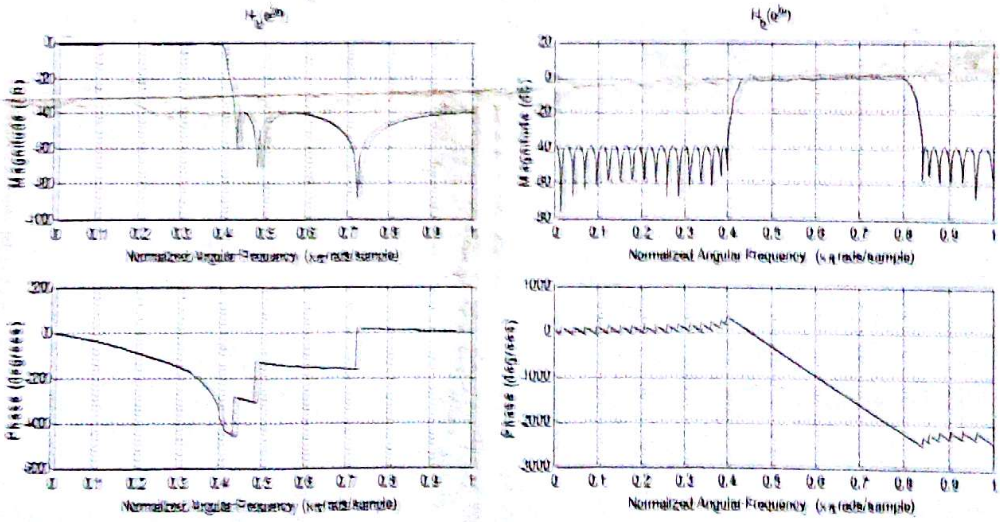
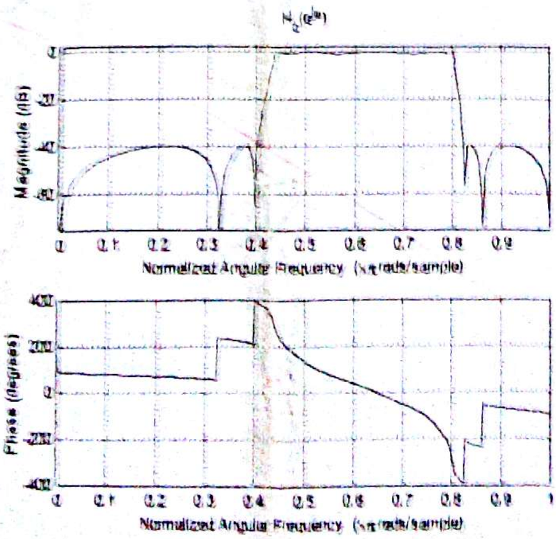

x(n)
y1(n)
y2(n)
y3(n)
Sample Index


Magnitude (dB)
Phase (degrees)
Normalized Angular Frequency (×π rads/sample)


Magnitude (dB)
Phase (degrees)
Normalized Angular Frequency (×π rads/sample)